Read this before starting!

- This test is closed book and closed notes
- You may **NOT** use a calculator
- All answers must have a box drawn around them. This is to aid the grader (who might not be me!) Failure to do so might result in no credit for answer.
- If you perform work on the back of a page in this test, indicate that you have done so in case the need arises for partial credit to be determined.
- Statement regarding academic misconduct from Section 5.7 of the East Tennessee State University Faculty Handbook, June 1, 2001:

  "Academic misconduct will be subject to disciplinary action. Any act of dishonesty in academic work constitutes academic misconduct. This includes plagiarism, the changing of falsifying of any academic documents or materials, cheating, and the giving or receiving of unauthorized aid in tests, examinations, or other assigned school work. Penalties for academic misconduct will vary with the seriousness of the offense and may include, but are not limited to: a grade of 'F' on the work in question, a grade of 'F' of the course, reprimand, probation, suspension, and expulsion. For a second academic offense the penalty is permanent expulsion."

**QUESTIONS BEGIN HERE!**

Problems 1, 2, and 3 represent relations across the Cartesian product $A \times A$ where $A = \{a, b, c, d\}$. The relations are represented either as subsets of $A \times A$, matrices, or digraphs. For each problem, determine whether the relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, and/or transitive. **Check all that apply. (3 points each)**

1. $R = A \times A$
   - reflexive
   - irreflexive
   - symmetric
   - asymmetric
   - antisymmetric
   - transitive

2. $R = \{(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, c), (d, d)\}$
   - reflexive
   - irreflexive
   - symmetric
   - asymmetric
   - antisymmetric
   - transitive

3. 
   \[
   \begin{bmatrix}
   0 & 0 & 1 & 1 \\
   1 & 0 & 1 & 1 \\
   0 & 0 & 0 & 1 \\
   0 & 0 & 0 & 0 
   \end{bmatrix}
   \]
   - reflexive
   - irreflexive
   - symmetric
   - asymmetric
   - antisymmetric
   - transitive
4. The digraph below represents a relation \( R \) on \( A = \{1, 2, 3, 4, 5\} \). Convert the digraph to a matrix. (3 points)

![Digraph](image)

5. Fill out the table below listing the in-degree and out-degree of each element for the relation of the previous problem. (3 points)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-Degree</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-Degree</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For problems 6, 7, and 8, let \( A = \{a, b, c, d\} \) and \( B = \{1, 2, 3, 4\} \). Determine whether the each of the relations \( R \) from \( A \) to \( B \) in these problems is a function. (2 points each)

6. \( R = A \times B \)
   - \( \square \) Function
   - \( \square \) Not a function

7. \( R = \{(a, 2), (c, 1), (d, 1), (b, 2)\} \)
   - \( \square \) Function
   - \( \square \) Not a function

8. \( R = \{(a, 4), (b, 3), (c, 2), (c, 1)\} \)
   - \( \square \) Function
   - \( \square \) Not a function

For problems 9 and 10, determine the domain and range of the function \( f \). In other words, if \( f(a) = b \), then what values of 'a' make sense for \( f \) (Dom(\( f \))) and what values of 'b' make sense for \( f \) (Ran(\( f \)))? By the way, please stick to subsets of real numbers. (3 points each)

9. \( f(a) = +\sqrt{a} \)
   - Dom(\( f \)) = ________________
   - Ran(\( f \)) = ________________

10. \( f(a) = a \mod 5 \) where \( a \) is an integer
    - Dom(\( f \)) = ________________
    - Ran(\( f \)) = ________________

For problems 11 and 12, let the universal set \( U = \mathbb{Z}^+ \) (the set of positive integers). Given the subset \( A \), determine the output of the given characteristic or membership function \( f_A \). (1 point each)

11. \( A = \{n \mid n = \text{even positive integer}\} \)
    - \( f_A(234) = ________ \)

12. \( A = \{0, 5, 10, 15, \ldots 5n\} \quad n = 0, 1, 2, \ldots \)
    - \( f_A(234) = ________ \)

For problems 13 and 14, let \( f \) be the mod-100 function. Compute the output for each of the problems. (2 points each)

13. \( f(34) = ________ \)

14. \( f(222) = ________ \)
Each relation \( R \) in problems 15 through 17 is defined on \( A = \{a, b, c, d, e\} \). In each case, determine if \( R \) is a rooted tree, and if it is, what is the root? If there is no root, leave that space blank. (3 points ea.)

15. \( R = \{(a, c), (a, d), (b, e), (b, c)\} \)
   \( \Box \quad R \) is a rooted tree \( \Box \quad R \) is not a rooted tree
   If \( R \) is a rooted tree, the root is: _____________

16. \( R = \{(c, a), (b, c), (e, d), (d, b)\} \)
   \( \Box \quad R \) is a rooted tree \( \Box \quad R \) is not a rooted tree
   If \( R \) is a rooted tree, the root is: _____________

17. \( R = \{(e, a), (e, c), (c, d), (c, b), (d, b)\} \)
   \( \Box \quad R \) is a rooted tree \( \Box \quad R \) is not a rooted tree
   If \( R \) is a rooted tree, the root is: _____________

For problems 18 through 24, use the rooted tree \( T \) shown in the figure to the right. (2 points each)

18. What is the height of \( T \)? _______________

19. \( T \) is an \( n \)-tree. What is the value of \( n \)? _______________

20. List all of the leaves of \( T \). _______________________

21. List all of the siblings of \( d \). _______________

22. List all of the offspring of \( d \). _______________

23. List all of the descendants of \( d \). ____________________

24. True or false: \( T \) is a complete \( n \)-tree? _______________

25. Construct the tree of the algebraic expression
   \((a \div 3) + 2) \times (b - 6)\). (4 points)
26. The following doubly linked list represents a binary positional labeled tree. Construct the digraph of this tree with each vertex labeled as indicated. (6 points)

<table>
<thead>
<tr>
<th>index</th>
<th>left</th>
<th>data</th>
<th>right</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>N</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>T</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>U</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>H</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>S</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>A</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>M</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>I</td>
<td>4</td>
</tr>
</tbody>
</table>

27. Fill in the LEFT and RIGHT arrays in the table to the left for the tree shown below. (6 points)

28. Use the Huffman code tree shown to the right to find the string of 0's and 1's that represents the word **PAYDAY**. (4 points)

29. Use the Huffman code tree shown to the right to decode the message **1110110001011111010**. (3 points)

30. The expression shown below is written in Polish (prefix) notation. Evaluate it to the final integer result. Note that all of the numbers are single digit integers. (3 points)

\[- \div \times 4 \ 2 \ - \ 6 \ 2 \ 1\]
31. The expression shown below is written in reverse Polish (postfix) notation. Evaluate it to the final integer result. Note that all of the numbers are single digit integers. (3 points)

\[ 5 \ 3 \ - \ 4 \ 5 \ + \ 3 \ \div \times \]

32. List the vertices in the order that they are visited in a preorder search of the tree shown to the right. (3 points)

[Diagram of a tree with vertices a, b, c, d, e, f, g, h, i]

33. List the vertices in the order that they are visited in an inorder search of the same tree from problem 32. (3 points)

34. In the space to the right, convert the tree shown below to a binary positional tree. (4 points)

[Diagram of a tree with vertices a, b, c, d, e]

35. Use any method you wish to determine the minimal spanning tree for the connected graph shown below and to the left. Draw the connections of the minimal spanning tree using the vertices shown to the right. (5 points)

[Diagram of a graph with vertices a, b, c, d, e, f, g, h, i]