Basic Rules of Boolean Algebra

<table>
<thead>
<tr>
<th></th>
<th>OR</th>
<th>AND</th>
<th>XOR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Combined w/0</strong></td>
<td>$A + 0 = A$</td>
<td>$A \cdot 0 = 0$</td>
<td>$A \oplus 0 = A$</td>
</tr>
<tr>
<td><strong>Combined w/1</strong></td>
<td>$A + 1 = 1$</td>
<td>$A \cdot 1 = A$</td>
<td>$A \oplus 1 = \overline{A}$</td>
</tr>
<tr>
<td><strong>Combined w/self</strong></td>
<td>$A + A = A$</td>
<td>$A \cdot A = A$</td>
<td>$A \oplus A = 0$</td>
</tr>
<tr>
<td><strong>Combined w/inverse</strong></td>
<td>$A + \overline{A} = 1$</td>
<td>$A \cdot \overline{A} = 0$</td>
<td>$A \oplus \overline{A} = 1$</td>
</tr>
</tbody>
</table>

**Other rules**

- $A + A \cdot B = A$
- $A + \overline{A} \cdot B = A + B$
- $(A + B) \cdot (A + C) = A + B \cdot C$

**DeMorgan's Th.**

- $(A \cdot B) = \overline{A + B}$
- $(A + B) = \overline{A} \cdot \overline{B}$
Short-ish Answer (2 points each unless otherwise noted)

1. Which unit of measurement is equivalent to (the same as) Hertz?
   a.) Cycles per second  
   b.) Percent  
   c.) Seconds  
   d.) Seconds per cycle  
   e.) Cycles

2. What is the frequency of the signal shown to the right?
   ![Signal Diagram]
   - 0.06 seconds
   - 0.14 seconds

3. The duty cycle for the previous problem is:
   a.) greater than 50%  
   b.) equal to 50%  
   c.) less than 50%

4. How many patterns of ones and zeros can be made using 9 bits?

5. What is the most negative value that can be stored using a 9-bit 2's complement representation?
   a.) \(-(2^7 - 1)\)  
   b.) \(-(2^8 - 1)\)  
   c.) \(-(2^9 - 1)\)  
   d.) \(-2^7\)  
   e.) \(-2^8\)  
   f.) \(-2^9\)

6. Gray code is:
   a.) a numbering system designed to best represent the color levels of a gray scale image
   b.) a representation of binary that allows for quick conversion to and from decimal
   c.) a secret language spoken only by people from Gray, TN
   d.) a binary representation meant to improve the speed with which data is stored to memory
   e.) a sequence of numbers where only a single bit changes when incrementing or decrementing through the sequence

7. For each of the following applications, what would be the optimum (best) binary representation, unsigned binary (UB), 2's complement (TC), IEEE 754 Floating Point (FP), or binary coded decimal (BCD)? Identify your answer in the blank to the left of each application. (2 points each)
   ________ the distance above (positive) or below (negative) sea level in feet to the nearest integer
   ________ the number of atoms in a grain of salt (a really huge number)
   ________ the value in dollars and cents of a financial portfolio

8. Write the complete truth table for a 2-input NOR gate.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>X</th>
</tr>
</thead>
</table>

9. In the boolean expression below, circle the operation that would be performed first.
   \[ A + B \cdot C \cdot D \]
10. Multiply the 16-bit value $0000110111000000_2$ by 8. **Leave your answer in 16-bit binary.** (Hint: Remember the shortcut!) (3 points)

11. Convert $101001001010101101_2$ to hexadecimal. (3 points)

12. Convert the decimal value $86_{10}$ to 8-bit BCD.

13. Convert the unsigned binary value $11001_2$ to its corresponding 5-bit binary Gray code. (3 points)

**Medium-ish Answer (4 points each unless otherwise noted)**

14. Convert the 32-bit IEEE 754 floating-point number $11000000110011100000000000000000$ to its binary exponential format, e.g., $1.1010110 \times 2^{-12}$, (which, by the way, is not even close to correct).

15. Convert $11001.101_2$ to decimal. (You may leave your answer in expanded form if you wish.)

16. Draw the circuit **exactly** as it is represented by the Boolean expression $\overline{A} + \overline{B} + A \cdot C$.

17. Prove that $A \oplus A = 0$. (Remember that $\oplus$ is the XOR or exclusive-OR)
18. Use any method you wish to prove the rule \( A + \overline{A} \cdot B = A + B \). Show all steps.

19. In the space to the right, create the truth table for the circuit shown below.

\[
\begin{array}{ccc|c}
A & B & C & X \\
\hline
& & & \\
\end{array}
\]

20. Write the Boolean expression for the circuit shown in the previous problem. *Do not simplify!*

**Longer Answers (Points vary per problem)**

21. Assume that an 8-bit binary number is used to represent an analog value in the range from 0 to 30. Convert all four of the following binary values to their analog equivalent, i.e., what analog value does each of these binary values represent? (You may leave your answer in the form of a fraction in some cases if you wish.) (5 points)

   a.) \( 00000000_2 \)

   b.) \( 00000001_2 \)

   c.) \( 00001010_2 \)

   d.) \( 11111111_2 \)
22. Use DeMorgan's Theorem to distribute the inverse of the expression $A + B + C + D$ all of the way to the individual input terms. *Do not simplify!*

23. Mark each Boolean expression as *true* or *false* depending on whether the right and left sides of the equal sign are equivalent. Show all of your work to receive partial credit for incorrect answers. (3 points each)

   a.) $(A + B) \cdot (\overline{A} + \overline{B}) = A \cdot \overline{B} + \overline{A} \cdot B$  
      Answer: ___________

   b.) $(A \cdot B + C) \cdot (A \cdot B + D) = A \cdot B + C \cdot D$  
      Answer: ___________

   c.) $A + A \cdot B + \overline{A} \cdot C + \overline{C} + A = A$  
      Answer: ___________
24. Fill in the blank cells of the table below with the correct numeric format. *For cells representing binary values, only 8-bit values are allowed!* If a value for a cell is invalid or cannot be represented in that format, write "X". (7 points per row)

<table>
<thead>
<tr>
<th>Decimal</th>
<th>2's complement binary</th>
<th>Signed magnitude binary</th>
<th>Unsigned binary</th>
<th>Unsigned BCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>01000100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>11000011</td>
<td></td>
<td></td>
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</tbody>
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